Implementing Meshes

Lecture 22

Robb T. Koether

Hampden-Sydney College

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Outline

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Contours

Definition (s- and t-Contours)

An s-contour is the 1-dimensional curve we get if we hold t fixed and let s vary over its domain. A t-contour is the 1-dimensional curve we get if we hold s fixed and let t vary over its domain.

- Typically, we get a different s-contour for each value of t and a different t-contour for each value of s.
- Together, the s- and t-contours form a grid, or a mesh, of the surface.

Outline

1. Contours
   - Contours of a Sphere
   - Contours of a Paraboloid
2. Calculating Normal Vectors
3. Assignment

Contours on a Sphere

Example (Contours on a Sphere)

- The sphere is paremetrized by

$$
\begin{aligned}
x &= \cos t \sin s \\
y &= \sin t \\
z &= \cos t \cos s.
\end{aligned}
$$

- Let $t = \frac{\pi}{3}$ and find the s-contour.

Contours on a Sphere

Example (Contours on a Sphere)

- We get

$$\begin{aligned} x &= \cos\tfrac{\pi}{3}\sin s = \tfrac{1}{2}\sin s \\ y &= \sin\tfrac{\pi}{3} = \tfrac{\sqrt{3}}{2} \\ z &= \cos\tfrac{\pi}{3}\cos s = \tfrac{1}{2}\cos s. \end{aligned}$$

- This is the circle

$$x^2 + z^2 = \frac{1}{4}$$

  at the height $y = \frac{\sqrt{3}}{2}$.

Contours on a Sphere

Example (Contours on a Sphere)

- Now let $s = \frac{\pi}{3}$ and find the t-contour.
- We get

$$
\begin{aligned}
x &= \cos t \sin \tfrac{\pi}{3} = \tfrac{\sqrt{3}}{2} \cos t \\
y &= \sin t \\
z &= \cos t \cos \tfrac{\pi}{3} = \tfrac{1}{2} \cos t.
\end{aligned}
$$

- This is a circle of radius 1 in the plane $x = \sqrt{3}z$.

Outline

Contours on a Paraboloid

Example (Contours on a Paraboloid)

- Find the s-contour of the paraboloid for $s = \frac{\pi}{3}$ and $t = \frac{1}{2}$.
- For the paraboloid,
  - Find the s-contour when $t = \frac{1}{2}$.
  - Find the t-contours when $s = 0$ and when $s = \frac{\pi}{2}$.

Outline

Finding Normal Vectors

- At a point $P(s, t)$,
  - A vector tangent to the s-contour is given by $\frac{\partial P}{\partial s}$.
  - A vector tangent to the t-contour is given by $\frac{\partial P}{\partial t}$.
  - These vectors lie in the tangent plane.
  - Thus, a normal vector to the surface is

$$\mathbf{n} = \frac{\partial P}{\partial s} \times \frac{\partial P}{\partial t}.$$

Finding Normals

Example (Finding Normals)

- Find the normals to the surface of the sphere defined by $x^2 + y^2 + z^2 = 1$.
- Let
$$P(s, t) = (\cos t \sin s, \sin t, \cos t \cos s).$$

Finding Normals

Example (Finding Normals)

- Then

$$\frac{\partial P}{\partial s} = (\cos t \cos s, 0, -\cos t \sin s)$$
$$\frac{\partial P}{\partial t} = (-\sin t \sin s, \cos t, -\sin t \cos s).$$

Finding Normals

Example (Finding Normals)

- Then

$$\begin{aligned}\mathbf{n} &= (\cos^2 t \sin s, \cos t \sin t \cos^2 s + \cos t \sin t \sin^2 s, \cos^2 t \cos s) \\ &= (\cos^2 t \sin s, \cos t \sin t + \cos t \sin t, \cos^2 t \cos s) \\ &= \cos t(\cos t \sin s, \sin t, \cos t \cos s) \\ &= \cos t(x, y, z) \\ \mathbf{N} &= \frac{\mathbf{n}}{|\mathbf{n}|} \\ &= (x, y, z).\end{aligned}$$

The Direction of the Normals

- Caution: This proceed is as likely to produce vectors that point "inward" as it is to produce vectors that point "outward."
- It depends on the parametrization.
- If the vectors point inward, then use the negative of the vector if you want them to point outward.

The Paraboloid Mesh

Example (The Paraboloid Mesh)

- Find the normal vectors for a paraboloid.
- Which way do they point?
- Which way should they point?